in L. Jacobs, A. Astudillo, J. Lataire, R. Sepulchre, J. Swevers, A. Pintelon, D. Peumans, W. Decré (Eds.),
34rd Benelux Meeting on Systems and Control, Blankenberge, Belgium, 2024

# Additive Continuous-time Identification: With Application to Modal Mechanical Systems

Rodrigo A. González[i,*], Koen Classens[i], Cristian R. Rojas[ii], James S. Welsh[iii] and Tom Oomen[i,iv][1]

## 1 Background

Parametric continuous-time identification applied to mechanical or biological systems yields interpretable models when the model structure aligns with the physical properties of the system. Traditional system identification may not consider the most parsimonious model structure when relying only on unfactored transfer functions, which result from standard direct approaches.

## 2 Problem Formulation

Consider the single-input single-output, linear and time-invariant, continuous-time system in additive form

$$x(t) = \sum_{i=1}^{K} G_i^*(p)u(t), \quad y(t_k) = x(t_k) + v(t_k),$$

where the numerator and denominator of each submodel $G_i^*(p)$ is described by the parameter vector $\theta_i^*$. We are concerned with designing data-driven methods to obtain estimates of $\beta^* := [\theta_1^{*\top}, \theta_2^{*\top}, \dots, \theta_K^{*\top}]^\top$ for the open- and closed-loop settings in Figure 1.

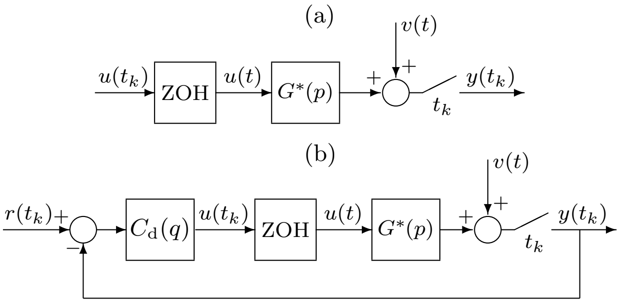

**Figure 1:** Block diagrams for the open (a) and closed-loop (b) settings studied in this work.

## 3 Approach: A Unified Estimator

In open-loop, identification can be posed as a first-order optimality condition; in closed loop, we employ an instrumental variable framework. Either way, these approaches can be shown to lead to

$$\hat{\beta} \in \operatorname*{sol}_{\beta\in\Omega} \left\{ \frac{1}{N}\sum_{k=1}^{N} \hat{\varphi}(t_k) \left( y(t_k) - \sum_{i=1}^{K} G_i(p,\theta_i)u(t_k) \right) = \mathbf{0} \right\},$$

where $\hat{\varphi}(t_k)$ is uncorrelated with the output noise $v(t_k)$. This work derives a vector $\hat{\varphi}(t_k)$ that minimizes the asymptotic covariance of the estimators in additive form for both open and closed-loop settings, and proposes an extension of refined instrumental variables for computing $\hat{\beta}$. The proposed iterations are given by

$$\mathcal{B}^{j+1} = \left[ \frac{1}{N}\sum_{k=1}^{N} \hat{\varphi}(t_k,\beta^j)\varphi^\top(t_k,\beta^j) \right]^{-1} \times \left[ \frac{1}{N}\sum_{k=1}^{N} \hat{\varphi}(t_k,\beta^j)\Upsilon^\top(t_k,\beta^j) \right],$$

where the next iteration $\beta^{j+1}$ is extracted from the block diagonal coefficients of $\mathcal{B}^{j+1}$, the regressor $\varphi$ is formed by the gradient of the model output with respect to $\beta$, and the instrument vector is the current estimate of the noise-free component of $\varphi$.

This estimator is shown to be generically consistent for both open and closed-loop settings, and can admit the identification of marginally stable additive systems.

## 4 Experimental Validation

We apply the proposed method to a slender and flexible steel beam suspended by wire flexures. The first four modes of the system are estimated (i.e., four second-order systems without zeros), see Figure 2. The proposed method converges to a parametric additive model closely aligned with the first four modes of the system.

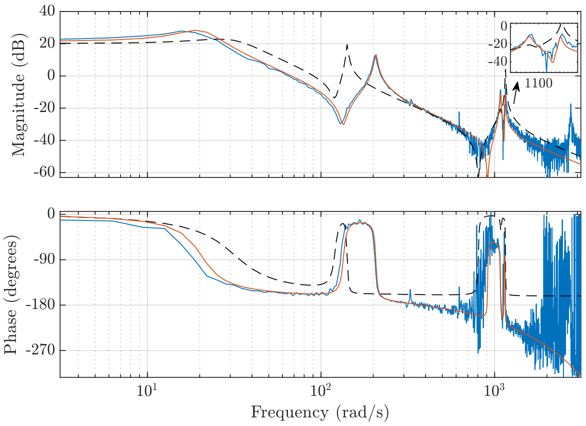

**Figure 2:** Estimation of the frequency response function of the flexible beam. Nonparametric (—), parametric modal (—), and initial (- -) estimates.

## References

[1] R.A. González, K. Classens, C.R. Rojas, J.S. Welsh and T. Oomen. "Identification of Additive Continuous-time Systems in Open and Closed-loop", *arXiv preprint 2401.01263*, 2024.

---

[1] This work was supported by the research program VIDI with project number 15698, with is (partly) financed by NWO.

[i] Control Systems Technology Section, Eindhoven University of Technology, The Netherlands. [ii] KTH Royal Institute of Technology, Stockholm, Sweden. [iii] University of Newcastle, NSW, Australia. [iv] Delft University of Technology, The Netherlands.

[*] Email: r.a.gonzalez@tue.nl.